B.Sc(H) Semester-I 2012/PH ②

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8826**

Unique Paper Code : **234103** C

Name of the Paper : **CSHT-102 Discrete Structures**

Name of the Course : **B.Sc. (Hons.) Computer Science Part I**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Parts of a question must be attempted together.

Part A (of **35** marks) is compulsory. Attempt any *four* questions from Part B.

Part A

(Compulsory)

1. (*a*) State whether Master's Theorem is applicable in each of the following. Solve the recurrence if it is applicable, else justify your answer :

   (*i*) $T(n) = 4T(n/2) + n^2 \lg n$

   (*ii*) $T(n) = 4T(n/2) + n$ 4

   (*b*) Show that 2^n is $O(3^n)$ but 3^n is not $O(2^n)$. 3

2. (*a*) Solve the recurrence relation. Find the total solution for the difference equation : 5

   $$a_r - 7\,a_{r-1} + 10\,a_{r-2} = 3^r, \text{ given that } a_0 = 0 \text{ and } a_1 = 1.$$

   (*b*) Determine the discrete numeric function corresponding to the generating function : 4

   $$A(z) = \frac{32 - 22z}{2 - 3z + z^2}.$$

3. (*a*) Among 50 students in a class, 26 got A in the Mathematics and 7 students got an A in both Mathematics and English. How many students did not get an A in English ? 3

(*b*) Find the reflexive closure of the following relation : 3

$$R = \{(3, 4), (4, 3), (4, 4), (5, 4)\} \text{ on the set } A = \{3, 4, 5\}.$$

(*c*) Suppose repetitions are not permitted : 3

(*i*) How many 3-digit numbers can be formed from the digits 1, 3, 5, 8, 9 ?

(*ii*) How many of them are between 4000 and 8000 ?

(*iii*) How many of them are even ?

4. (*a*) Show that $\neg P$ is tautologically implied by $\neg (P \wedge \neg Q)$, $(\neg Q \vee R)$, $\neg R$. 4

(*b*) Draw an Acquaintanceship graph of 7 students in a college. Assume that each student knows atmost 3 and at least 1 student. What do the vertices and edges represent ? 3

(*c*) Find the number of edges in a full binary tree with 50 internal nodes. 3

Part B

(Attempt any *four* questions from Part B)

5. (*a*) Prove by Mathematical Induction that, for $n \geq 1$: 3

$$1.2 + 2.3 + \ldots\ldots\ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

(*b*) Given $A = \{a, b, c, d\}$. Consider the relation R on A :

$$R = \{(a, a), (b, b), (b, c), (c, b), (d, b), (d, d)\}$$

Is R Symmetric ? Is it Transitive ? Give reasons. 4

(*c*) Assume that classes are not held on weekends, show that in a set of six classes there must be two that meet on the same day. Justify your answer using the Pigeonhole principle. 3

6. (*a*) Determine whether each of these functions from {1, 2, 3, 4} to itself is one-to-one and/or onto :

(*i*) $f(1) = 2,\ f(2) = 1,\ f(3) = 3,\ f(4) = 4$

(*ii*) $f(1) = 4,\ f(2) = 2,\ f(3) = 4,\ f(4) = 3.$ 5

(*b*) In how many ways can 2 numbers be selected from the integers 1-100, so that their sum is an even number ? 2

(*c*) Show that $C(2n, 2) = 2\,C(n, 2) + n^2$. 3

7. (*a*) Show the equivalence :

$$(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q).$$ 4

(*b*) Consider the predicate $P(x)$: x is less than equal to 4. Which of the following statements would be true for the universe of discourse {–2, –1, 3, 4, 5, 6} ? How ?

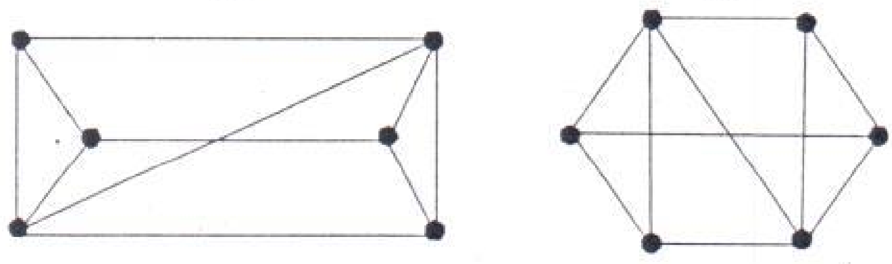
(*i*) $(\exists x)\ P(x)$

(*ii*) $(x)P(x)$. 4

(*c*) Given the value of $P \rightarrow Q$ is false, determine the value of $(\neg P \vee \neg Q) \rightarrow Q$. 2

8. (*a*) Are the graphs G1 and G2 isomorphic ? Explain. 4

G1 G2



(*b*) Prove the Euler's formula for Planar graphs. 4

(*c*) Define a spanning tree of a graph. Draw a spanning tree of the graph G1 of Q. No. 8(*a*). [2]

P.T.O.

9. (*a*) Simplify the expression $\Sigma_{k=1}^{n} (2k - 1)$. 3

(*b*) Use Bubble Sort to sort the list 3, 1, 5, 7, 4. Show the list obtained at each step. 4

(*c*) Show that $3x^2 + x + 1$ is $\Theta(x^2)$, by finding the constants k, C_1, C_2. 3

10. (*a*) Determine $a * b$ in the following : 3

$$a_r = \begin{cases} 1, & 0 \le r \le 2 \\ 0, & r \ge 3 \end{cases} \quad b_r = \begin{cases} 1, & 0 \le r \le 2 \\ 0, & r \ge 3 \end{cases}$$

(*b*) Use Substitution Method to show that the solution of $T(n) = T(\lfloor n/2 \rfloor) + 1$ is $O(\lg n)$. 3

(*c*) Determine the particular solution for the difference equation : 4

$$a_r - 4a_{r-1} + 4\, a_{r-2} = (r + 1) \,.\, 2^r.$$